Lab Exercise #9: Combined Loading

Pre-lab assignment: ☑ Yes  ☐ No

Goals:
- To determine principal stresses and strains using a rosette strain gage and compare with theoretical values.

Principles:
Previous labs have presented various loading scenarios which have lead to multiple states of stress. In the beginning, tensile and bending tests were performed to illustrate the principles of uniaxial stress. In tension or compression, an element experiences normal stress along a single axis and an absence of any shearing stress, as seen in Fig. 1a. In a similar manner, 4-point bending tests revealed a very similar state of stress. Even though the type of loading is different, the development of normal stress (without shearing stress) is uniaxial and very similar. Of course, depending on where an element is located within a beam in bending, the stress may be tensile or compressive.

After observing the effects of normal stress during tension and bending, shear stress was then observed while loading small rods in torsion. The presence of torque, or twisting action, on a rod creates shear stress on the elements within the rod. This state of stress is seen in Fig. 1b. The elements located on the surface of a rod undergoing torque experience the largest magnitude of shear stress.

Yet another experiment introduced biaxial stress by studying the hoop stress and longitudinal stress that is present on the elements of a pressure vessel. Although biaxial stress is technically a type of plane stress, it is a simplified scenario since there is no shearing action. With biaxial stress, an element experiences normal stress on perpendicular axes, shown in Fig. 1c.

![Figure 1](image)

In engineering applications, it is very common to encounter loading conditions that create states of stress with both normal and shear stress together. This combined loading is typical of shafts that are undergoing flexure (bending), shear and torsion. For example, a shaft that is transmitting power from a motor to a transmission may experience such conditions. Any point on the surface of that shaft will experience a state of plane stress that may be similar to Fig. 1d or Fig. 1e. In either case, both normal and shear stresses are present.

In the first three cases of Figure 1, it is relatively simple to calculate the principal stresses. In (a) and (c), they are determined by inspection since they are equal to the applied normal stresses in the horizontal and vertical directions. In the case of pure shear, the principal stresses are of equal magnitude to the shear stress and oriented at 45° and 135° from horizontal.

In the case of plane stress, as in (d) and (e), it requires the use of principal stress equations or Mohr’s circle to determine the magnitude and angle of the principle stresses.
The purpose of this experiment is to calculate the principal stresses on a cantilevered beam experiencing combinations of flexural, shear, and torsional stress (see Fig. 2). Points on the surface of the beam will experience a state of plane stress. However, as with previous labs it is not possible to directly measure the applied stresses. Instead, strain gages will be used to measure strain values. The use of a strain transformation equation and Hooke’s law will allow for the calculation of stress.

Before describing the procedural steps for the experiment, it should be noted that several combinations of stress states can be achieved by loading the cantilevered beam and moment arm in various ways. The descriptions below provide details specific for an element located on the top of the cantilevered beam at *Point A* in Fig. 2.

**Normal (flexural) stress only** – to create a uniaxial state of stress at *Point A*, a weight can be hung at position 1 at the end of the beam. This will create bending and shear in the beam. The bending creates a flexural stress at *A*. However, even though a vertical shear force is present, shear stress is not present because *A* is located at the top of the beam (and *Q* is zero at the top).

**Torsional stress only** – in order to create a shear stress due to torsion at *A*, a weight can be hung at position 2. However, this will also create a bending moment. In order to counteract the bending tendency, a force of equal value must be applied at position 3 in the upward direction. In this scenario, there will be no vertical shear force in the beam.

**Combined stress** – if a weight is hung at position 2, it will have the combined effect of creating shear stress due to torsion and flexural (normal) stress at *A*.

This experiment will evaluate the stress state at *Points A* and *B* by measuring the strain at each location with a strain rosette. Figure 3 shows a strain rosette, which is essentially three strain gages located very close together. It must be recognized that the 2D state of strain at a point on the surface is defined by three independent quantities, $e_x$, $e_y$ and $\gamma_{xy}$. A method does not exist to determine $\gamma_{xy}$ directly. However, if three independent measurements of strain are taken as close as possible to a point, the use of strain transformation equations can be used to determine $e_x$, $e_y$ and $\gamma_{xy}$. As an example, the rosette in Fig. 3 is placed on the surface of a beam.

If $e_x = e_a$ and $e_y = e_c$:

\[
\theta = \frac{e_a - e_c}{2e_a} \quad (1)
\]

In this equation, $\theta$ is the angle between the horizontal gage and the angled gage. For the rosettes in this experiment, $\theta = 45^\circ$. With the information above, $e_x$, $e_y$ and $\gamma_{xy}$ can be determined. For more information, see the section titled “Calculation of Stresses from the Strains” at the end of 7.7 in Gere. The next step will be to find the stresses that correspond to the $x$ and $y$ axes seen in Fig. 3. Equations 2 and 3 are a result of Hooke’s Law for plane stress and allow for the calculation of $\sigma_x$ and $\sigma_y$. Equation 4 is used to determine $\tau_{xy}$.
Finally, the principle stresses, maximum shear stress, and principle planes can be determined by use of either transformation equations or Mohr’s circle.

Materials:
- 4 ft. aluminum (2024 T3) cantilever beam with attached moment arm
  - Beam is equipped with two 0-45-90° strain rosettes (“rectangular rosette”)
- Loading hangers and weights
- Strain gage reader

Safety Issues:
Weights will be applied to hangers to develop bending moment and torque loading on the cantilever beam. The weights may hang over the end of a table. Use caution when loading the weights. Always be aware that the weights may fall due to unbalanced loading, a broken string, or other unforeseen cause. Keep feet and hands away from the area under the weights.

Pre-lab:
Refer to the Pre-Lab Worksheet for details on exercises to be completed before lab. In order to complete Part II, it may be necessary to review the orientation and direction of shear forces (from torsion and shear) in Sections 3.5, 4.3, and 5.8.

Procedure:
In order to complete the exercise, the cantilevered beam will be loaded in various ways for Points A and B according to Fig. 2. See the Data Sheet for details on loads and positions.

- **Point A**
  - **Bending only** – apply loads at position 1 only.
  - **Torsion only** – apply loads at position 2. Counterbalance each load with an equivalent force at position 3 (will be done with a weight and pulley).
  - **Combined loading** – apply weights at position 2.

- **Point B**
  - **Combined loading** – apply weights at position 2.

Data Analysis (Use SI units):
Note: Due to the large number of calculations, it is important that Excel be used to repeat similar calculations. Use the provided Excel template file.

For **Point A**
1. Using the strain gage data, determine $\varepsilon_x$, $\varepsilon_y$, and $\gamma_{xy}$ for each loading condition.
   a. Calculate $\sigma_x$, $\sigma_y$, and $\tau_{xy}$.
   b. Calculate principle stresses $\sigma_1$ and $\sigma_2$ as well as principle directions $(\theta_{p1}, \theta_{p2})$.
2. For each of the the 40 N loading conditions, calculate the theoretical values of $\sigma_x$, $\sigma_y$, $\tau_{xy}$, $\sigma_1$, $\sigma_2$, and $\theta_{p1}$, $\theta_{p2}$.
For **Point B**

1. Using the strain gage data, determine $\varepsilon_x$, $\varepsilon_y$ and $\gamma_{xy}$ for the **combined loading case only**.
   
   a. Calculate $\sigma_x$, $\sigma_y$, and $\tau_{xy}$.
   
   b. Calculate principle stresses $\sigma_1$ and $\sigma_2$ as well as principle directions $(\theta_{p1}, \theta_{p2})$.

2. For the 40 N loading condition, calculate the theoretical values of $\sigma_x$, $\sigma_y$, $\tau_{xy}$, $\sigma_1$, $\sigma_2$, and $\theta_{p1}$, $\theta_{p2}$.

**Lab Report**

A lab report is **not** required for this exercise. The teaching assistant will explain the requirements for receiving credit.
Part I
Determine the following mechanical properties for 2024 T3 aluminum. Use SI units. Refer to a website such as matweb.com if your textbook does not have the information.

\[ E = \underline{\text{________________________}} \]

\[ \nu = \underline{\text{________________________}} \]

\[ G = \underline{\text{________________________}} \]

Calculate the area moment of inertia and polar moment of inertia (through the center) for a hollow round tube with the diameters below. Use SI units.

Outside diameter: 5.08 cm (2”)
Inside diameter: 4.83 cm

\[ I = \underline{\text{________________________}} \]

\[ J = \underline{\text{________________________}} \]

In your textbook, find the applicable formula for determining the maximum shear stress in a hollow tube beam. (Find the formula for shear stress due to shear forces, not from torsion.) Write the formula below.

Part II
A portion of the beam is shown from the top at Point A. For the combined loading scenario seen in Fig. 4, draw the flexural and shear stresses in the correct direction on the (oversized) element at A. Label the stresses. Figure 5 is given as an example of what is expected.

Repeat for Point B.
Bending Only

<table>
<thead>
<tr>
<th>Weight* (N)</th>
<th>( \varepsilon_a )</th>
<th>( \varepsilon_b )</th>
<th>( \varepsilon_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Torque Only (must apply equivalent counterweight)

<table>
<thead>
<tr>
<th>Weight* (N)</th>
<th>( \varepsilon_a )</th>
<th>( \varepsilon_b )</th>
<th>( \varepsilon_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Combined loading

<table>
<thead>
<tr>
<th>Weight* (N)</th>
<th>( \varepsilon_a )</th>
<th>( \varepsilon_b )</th>
<th>( \varepsilon_c )</th>
<th>( \varepsilon_{\text{a}} )</th>
<th>( \varepsilon_{\text{b}} )</th>
<th>( \varepsilon_{\text{c}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* The weight of the hangers can be disregarded.

Strain gage labels

Measure the dimensions shown (in cm)